INTEGRAL TRANSFORM TECHNIQUE OF SELF-TERM WIRE ANTENNA KERNEL

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1 Introduction

The geometry of a cylindrical antenna with length h and radius a is shown in Figure 1. The general self-interaction term for evaluating the magnetic vector and electric scalar potentials due to a $\hat{z}$-directed uniform current can be rewritten, which takes the form of (4) in [1]

$$\Im = \frac{1}{2\pi}\int_0^{\Delta}\int_{\phi'=0}^{2\pi}\frac{e^{-j\beta R_s}}{R_s}\,d\phi' dz' \tag{1}$$

with

$$R_s = \sqrt{z'^2 + 4a^2\sin^2(\phi'/2)} \tag{2}$$

where 2Δ and a represent one segment length and radius of wire antenna, respectively. The geometry of a cylindrical antenna with length h and radius a is shown in Figure 1.

The application of standard quadrature rules to integral (1) whose integrand possesses a singularity requires considerable computational time. One possible way to overcome this problem is to subtract out the singularities. Thus, subtracting out the singularity leads to

$$\Im = \Im_0 + \Im_1 = \frac{1}{2\pi}\int_0^{\Delta}\int_{\phi'=0}^{2\pi}\left\{\frac{1}{R_s} + \frac{e^{-j\beta R_s}-1}{R_s}\right\}d\phi' dz'. \tag{3}$$

$\Im_0$ and $\Im_1$ could be defined as

$$\Im_0 = \frac{1}{2\pi}\int_0^{\Delta}\int_{\phi'=0}^{2\pi}\frac{1}{R_s}\,d\phi' dz' \tag{4}$$

$$\Im_1 = \frac{1}{2\pi}\int_0^{\Delta}\int_{\phi'=0}^{2\pi}\left\{\frac{e^{-j\beta R_s}-1}{R_s}\right\}d\phi' dz' \tag{5}$$

where the second term of the integrand in (3) has a slowly varying function.

For this reason, this part can be easily evaluated with standard numerical integration. This paper presents another new integral transform technique for evaluating the double integral $\Im_0$ of (4), which greatly facilitates the numerical integration with accuracy and efficiency.

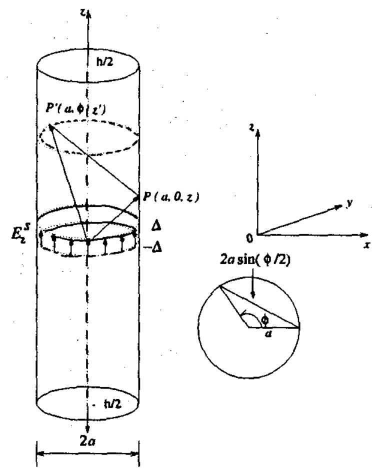

Figure 1: Geometry of cylindrical antenna

2 Integral Transform Technique of a Singular Kernel

To evaluate the double integral of (4) on the surface of the cylinder, $\Im_0$ can be rewritten in the form (eq. 6, in [1])

$$\begin{aligned} \Im_0 &= \frac{1}{2\pi}\int_0^{\Delta}\int_{\phi'=0}^{2\pi}\frac{d\phi' dz'}{\sqrt{z'^2+4a^2\sin^2(\phi'/2)}} \\ &= \frac{2}{\pi}\int_0^{\Delta}\int_0^{\infty}\cos kz' I_0(ka)K_0(ka)\,dk dz'. \end{aligned} \tag{6}$$

where $I_m(ka)$ and $K_m(ka)$ are modified Bessel functions of the first and second kind, respectively. Upon integration with respect to z', (6) reduces to

$$\Im_0 = \frac{2}{\pi}\int_0^{\infty}\frac{\sin\Delta k}{k}K_0(ak)I_0(ak)\,dk. \tag{7}$$

After modifying $K_0(ak)\cdot I_0(ak)$, equation (7) can be expressed as

$$\Im_0 = \frac{2}{\pi}\int_0^{\infty}[J_0(a\eta)]^2\,\eta\int_0^{\infty}\frac{\sin\Delta k}{(\eta^2+k^2)\,k}dk d\eta. \tag{8}$$

The infinite double integral of (8) can be further simplified if one of the integration has an analytic solution. Using the formula 3.725.1 of [2], the infinite double integral (8) can be converted into only an infinite one dimensional integral as

$$\Im_0 = \int_0^{\infty}[J_0(a\eta)]^2\cdot\left\{\frac{1-e^{-\Delta\eta}}{\eta}\right\}d\eta. \tag{9}$$

However, the obtained infinite integral formula of (9) is not suitable for evaluating numerical integration. The integral (9) over the η plane can be converted into an integration over the x plane by using Parseval's theorem

$$\int_0^{\infty}F_1(\eta)\,F_2(\eta)\,d\eta = 2\pi\int_0^{\infty}f_1(x)\,f_2(x)\,dx \tag{10}$$

since $F_1(\eta)$ and $F_2(\eta)$ are an even function of η. Let us define $F_1(\eta) = [J_0(a\eta)]^2$ and

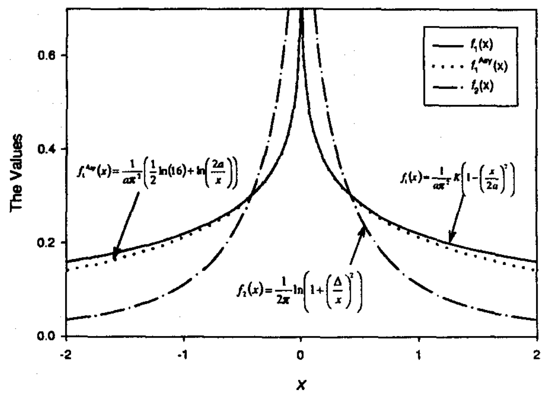

Figure 2: The values of $f_1(x), f_2(x)$, and $f_1(x)^{Asy}$ at $\Delta = a = 1$.

$F_2(\eta) = \left(1 - e^{-\Delta\eta}\right)/\eta$. Deduced from formula 6.672.6 in [2], $f_1(x)$ can be solved as

$$\begin{aligned} f_1(x) &= \frac{1}{\pi}\int_0^\infty [J_0(a\eta)]^2 \cos(\eta x)\, d\eta \\ &= \begin{cases} \left(\frac{1}{a\pi^2}\right) Q_{-\frac{1}{2}}\left(1 - 2\left(\frac{x}{2a}\right)^2\right), & \text{if } \ 0 < |x| < 2a \\ 0, & \text{if } \ |x| > 2a \end{cases} . \end{aligned} \tag{11}$$

where $Q_{-\frac{1}{2}}(x)$ is referred to as a spherical Legendre function of the second kind. With the aid of the formula 3.951.3 in [2], $f_2(x)$ can be easily obtained as

$$\begin{aligned} f_2(x) &= \frac{1}{\pi}\int_0^\infty \left[\frac{1 - e^{-\Delta\eta}}{\eta}\right] \cdot \cos(\eta x) d\eta \\ &= \frac{1}{2\pi}\ln\left(1 + \left(\frac{\Delta}{x}\right)^2\right). \end{aligned} \tag{12}$$

Substituting (11) and (12) into (10), the infinite integral of (9) with respect to η results into the finite integral explicitly as

$$\Im_0 = \frac{1}{a\pi^2}\int_0^{2a} K\left[1 - \left(\frac{x}{2a}\right)\right] \cdot \ln\left[1 + \left(\frac{\Delta}{x}\right)^2\right] dx. \tag{13}$$

where $K(m)$ is the complete elliptical integral of the first kind.

The integrand of (13) contains an integrable singularity. In order to evaluate the integral (13), we subdivide the interval of integration into the two regions; the region at and near the singularities($[0, \delta]$), and the remaining region away from the singularity($[\delta, 2a]$) as follows;

$$\int_0^{2a} f_1(x) \cdot f_2(x)\, dx = \int_0^{\delta} \left[f_1(x)^{Asy} \cdot f_2(x)^{Asy}\right] dx + \int_\delta^{2a} [f_1(x) \cdot f_2(x)]\, dx. \tag{14}$$

A direct integration of the first integral in (14) gives an analytical solution. Since the function $f_1(x)$ and $f_2(x)$ are well behaved in the regions($[\delta, 2a]$), the second integral of (14) is carried out numerically by using the self-adaptive integration scheme. The values of $f_1(x), f_2(x)$, and f_1^{Asy} are evaluated at the parameters of $\Delta = a = 1$, and plotted in Figure 2.

Table I: The integration values of $\Im_0$ by using (13) and other methods.

$\frac{\Delta}{a}$	Butler's Series (10) in [3]	(10*a*) in [1]	MININEC in [4]	The Proposed Method
0.1		0.171	0.171	0.171
0.5		0.599	0.599	0.599
0.9		0.908	0.905	0.905
1.3			1.150	1.150
1.7			1.354	1.354
2.0	1.487		1.487	1.487
2.3	1.605		1.605	1.605

3 Comparisons and Conclusions

The integration of (13), which is valid for the entire region of Δ/a, was performed and tabulated in on the fourth column of Table I. To check the validity of the one dimensional integral of (13), the values of other three different methods are evaluated and listed in Table I. First, Butler's series form (10) in [3] is listed on the first column of Table I. The series solution of (10a) in [1] is listed on the second column of Table I.

The singular treatment used in MININEC in [4], which is valid for the entire region of Δ/a, was also evaluated and listed on the third column of Table I. As can be seen in Table I, the proposed method has an excellent agreement with other three comparison results across the entire range of Δ/a. These results demonstrate that the newly derived transformed 1-D integral of (13) is valid without limitation.

References

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