RCS is shown in Figure 2. As expected, these results correspond with the measured results in [4].

4. CONCLUSION
This letter presents an alternative implementation of TD-AIM for solving scattering problems. Since the cubic spline is as the temporal basis function and the rot operator from the integrals of far-zone interactions is analytically removed, the interpolating procedure both for temporal and space in TD-AIM is simple and accurate, and the TD-CFIE can be solved with relatively littler memory and computational resources accurately. Simulation results have been presented. In our experience, when $N_{FFT} = 3$, the computing speed of the largest FFT is acceptable.

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Figure 2  Bistatic RCS (VV) of the NASA almond in the x-z plane $f = 4$ GHz

ADI-FDTD METHOD PERTURBED BY THE SECOND ORDER CROSS DERIVATIVE TERMS

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ABSTRACT: A two-step FDTD method as a compromise of conditional stability and reduced splitting error is formulated and its numerical stability is investigated. It is the perturbed form to the ADI-FDTD method by the addition of second order cross derivative term. It is validated from the comparison of numerical anisotropy and numerical error over the ADI-FDTD that numerical performances can be improved by controlling the perturbed term within the stable region of the cross derivative term. © 2008 Wiley Periodicals, Inc. Microwave Opt Technol Lett 50: 1822–1826, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23479

Key words: FDTD; ADI-FDTD; CFL condition; truncation error; dispersion

1. INTRODUCTION
The finite-difference time-domain (FDTD) method has been intensively used for many electromagnetic applications due to its low computational complexity, great flexibility, easy implementation, and so on. However, its time step is limited to the Courant-Friedrich-Lewy (CFL) stability condition, resulting from its explicit time-stepping. To remove the CFL stability condition, an implicit alternating-direction-implicit (ADI) FDTD algorithm was proposed by Namiki [1]. Although the ADI-FDTD method produces only a small computational load by its tridiagonal feature compared with the numerically inefficient Crank Nicolson (CN)-FDTD, it has large numerical dispersion error, especially in sharply changing regions, with large time steps due to its truncation term in its two-step factorization [2]. To be worse, since the numerical phase velocity is a function of the propagating directions, the cell-based medium becomes anisotropic. This anisotropy generates a direction-dependent phase error, so having difficulties to remedy the numerical dispersion error. These errors are accumulated as the numerical waves propagate, decreasing the numerical accuracy for solving some problems, for example, electrically large objects [3].

To reduce the numerical anisotropy, the optimization parameter [4] and the artificial anisotropy [5] were introduced into the ADI-FDTD formulation. However, the anisotropy still exists except in the case of zero dispersion in the chosen angles.

In this Letter, a two-step FDTD scheme based on the efficient two-step formulation is presented to reduce the numerical anisotropy by the addition of second-order cross derivative term, the perturbed form to the ADI-FDTD. The optimized method was found to improve numerical performances over a wider bandwidth compared with the ADI-FDTD algorithm.

2. DERIVATION OF THE PROPOSED ADI-FDTD METHOD
The 2D Maxwell’s equations for the TE-z case can be written as the following matrix system separated by two matrices of $x$ and $y$ variables, respectively.

$$\frac{\partial}{\partial t} \mathbf{U} = \mathbf{A} \mathbf{U} + \mathbf{B} \mathbf{U} \quad (1)$$
where \( \tilde{U} = [E, E, H, J]^T \).

\[
A = \begin{bmatrix}
0 & -\frac{\partial}{\varepsilon\partial y} & 0 & 0 \\
0 & 0 & 0 & -\frac{\partial}{\varepsilon\partial x} \\
\frac{\partial}{\mu\partial y} & 0 & 0 & 0 \\
\frac{\partial}{\mu\partial x} & 0 & 0 & 0
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\partial}{\varepsilon\partial x} \\
-\frac{\partial}{\mu\partial y} & 0 & 0 & 0 \\
-\frac{\partial}{\mu\partial x} & 0 & 0 & 0
\end{bmatrix}
\]

Here, \( \varepsilon \) and \( \mu \) are the permittivity and permeability in free space. Replacing the time derivative of (1) with its corresponding difference operation generates the following CN-FDTD formulation.

\[
(I - \frac{\Delta t}{2} A - \frac{\Delta t}{2} B) \tilde{U}^{n+1} = (I + \frac{\Delta t}{2} A + \frac{\Delta t}{2} B) \tilde{U}^n
\]

(2)

where \( I \) denotes the identity matrix of the 3 \times 3 dimension. While (2) can be solved by means of Gaussian elimination, it is inefficient due to its inherently time-consuming computation process. The CN-FDTD difference equation can be exactly changed to the following two equations from the Peaceman-Rachford scheme [6].

\[
\begin{align*}
(I - \frac{\Delta t}{2} A) \tilde{U}^{n+1/2} &= \left[ I + \frac{\Delta t}{2} B \right] \tilde{U}^n + \frac{\Delta t^2}{8} AB(\tilde{U}^{n+1} - \tilde{U}^n) \\
(I - \frac{\Delta t}{2} B) \tilde{U}^{n+1} &= \left[ I + \frac{\Delta t}{2} A \right] \tilde{U}^{n+1/2} + \frac{\Delta t^2}{8} AB(\tilde{U}^{n+1} - \tilde{U}^n)
\end{align*}
\]

(3a, 3b)

Truncating the last second-order terms of time-step size can generate the ADI-FDTD formulation. The effect of this truncation error can be classified in three factors: the time-step size \( (\Delta t) \), the spatial derivatives of the field \( (AB) \), and the temporal variation \( (\tilde{U}^{n+1} - \tilde{U}^n) \). The splitting error associated with the square of the time-step size becomes dominant in regions with larger spatial derivatives.

The difference Eq. (3) can be rewritten as,

\[
\begin{align*}
(I - \frac{\Delta t}{2} A - \frac{\Delta t^2}{8} AB) \tilde{U}^{n+1/2} &= \left[ I + \frac{\Delta t}{2} B - \frac{\Delta t^2}{8} AB \right] \tilde{U}^n + \frac{\Delta t^2}{8} AB(\tilde{U}^{n+1} - \tilde{U}^{n+1/2}) \\
(I - \frac{\Delta t}{2} B - \frac{\Delta t^2}{8} AB) \tilde{U}^{n+1} &= \left[ I + \frac{\Delta t}{2} A - \frac{\Delta t^2}{8} AB \right] \tilde{U}^{n+1/2} + \frac{\Delta t^2}{8} AB(\tilde{U}^{n+1} - \tilde{U}^n)
\end{align*}
\]

(4a, 4b)

Ignoring the last terms of (4) can be reduced into an efficient two-step formulation with the second-order cross derivative term included, just like in the ADI fashion of (3).

\[
\begin{align*}
(I - \frac{\Delta t}{2} A - \frac{\Delta t^2}{8} AB) \tilde{U}^{n+1/2} &= \left[ I + \frac{\Delta t}{2} B - \frac{\Delta t^2}{8} AB \right] \tilde{U}^n \\
(I - \frac{\Delta t}{2} B - \frac{\Delta t^2}{8} AB) \tilde{U}^{n+1} &= \left[ I + \frac{\Delta t}{2} A - \frac{\Delta t^2}{8} AB \right] \tilde{U}^{n+1/2}
\end{align*}
\]

(5a, 5b)

Solving (5) generates lower numerical error than the ADI-FDTD formulation due to discarding only half size \( (\tilde{U}^{n+1} - \tilde{U}^{n+1/2}) \) of the temporal variation in the last truncation terms in (4) instead of the full size \( (\tilde{U}^{n+1} - \tilde{U}^n) \) in the last truncation terms of (3) in the ADI-FDTD method. Moreover, we will introduce a degree of freedom by adding a parameter \( \alpha \) in the square term of time step in such a way to control numerical anisotropy as below,

\[
\begin{align*}
(I - \frac{\Delta t}{2} A - \alpha \Delta t^2 AB) \tilde{U}^{n+1/2} &= \left[ I + \frac{\Delta t}{2} B - \alpha \Delta t^2 AB \right] \tilde{U}^n \\
(I - \frac{\Delta t}{2} B - \alpha \Delta t^2 AB) \tilde{U}^{n+1} &= \left[ I + \frac{\Delta t}{2} A - \alpha \Delta t^2 AB \right] \tilde{U}^{n+1/2}
\end{align*}
\]

(6a, 6b)

The first procedure of (6-a) is described as follows,

\[
E_{n+1/2}^{x,x+1,j} - E_{n+1/2}^{x,j} = \frac{\Delta t}{2\varepsilon\Delta x} (H_{n+1}^{x+1,j+1/2,j} - H_{n+1}^{x+1,j,j+1/2})
\]

- \( \frac{\Delta t}{\varepsilon\mu\Delta y} (E_{n+1/2}^{x+1,j+1/2,j} - E_{n+1/2}^{x+1,j+1/2,j+1} + E_{n+1/2}^{x+1,j+1/2,j+1} - E_{n+1/2}^{x+1,j+1/2,j+1})
\]

(7)

\[
H_{n+1/2}^{x,x+1,j} - H_{n+1/2}^{x,j} = \frac{\Delta t}{2\mu\Delta x} (E_{n+1}^{x+1/2,j+1,j+1/2} - E_{n+1}^{x+1/2,j+1,j+1/2})
\]

(8)

\[
E_{n+1/2}^{x+1,j+1} - E_{n+1/2}^{x,j+1} = -\frac{\Delta t}{2\varepsilon\Delta x} (H_{n+1}^{x+1,j+1/2,j+1} - H_{n+1}^{x+1,j+1/2,j+1})
\]

(9)

where \( \Delta x \) and \( \Delta y \) are the spatial step sizes. In the first procedure, since (7) and (9) cannot be calculated directly, \( (7)' \) is derived from (7) and (9) by eliminating the \( H \) components as follows,

\[
\begin{align*}
-\frac{\Delta t^2}{4\varepsilon\mu\Delta y^2} E_{n+1/2}^{x,x+1,j+1} + \left( 1 + \frac{\Delta t^2}{2\varepsilon\mu\Delta y} \right) \cdot E_{n+1/2}^{x,j+1/2,j+1} \\
-\frac{\Delta t^2}{4\varepsilon\mu\Delta x^2} E_{n+1/2}^{x+1,j+1/2,j} - E_{n+1/2}^{x+1,j+1/2,j+1} = E_{n+1}^{x+1/2,j+1,j+1/2}
\end{align*}
\]

(7')

By simultaneously solving the linear Eq. (7)' and (8), we can get the values of the electric-field components at the \( (n + 1/2) \) time step. Then, we can obtain the magnetic-field components at \( (n + 1/2) \) from (9). The \( E \) term only include the square terms of time step multiplied by the mixed derivatives of the \( x \) and \( y \) spatial variables. The second procedure of (6-b) can be expanded, similarly to (6-a). Figure 1 summarizes the numerical calculation process of the proposed method. The numerical stability of (6) can be analyzed by Fourier analysis method. A 2D TE-z wave of the first procedure is defined as follows:

\[
\psi = \psi_0 \xi \exp(j(k_x x + k_y y))
\]

(10)

where \( j = \sqrt{-1} \), \( k_x \), and \( k_y \) are wavenumbers along the \( x \) and \( y \) axes, and \( \xi \) indicates the amplification factor of the first proce-

The amplification factor of a full iteration period can be found as,

\[ -1 - \sqrt{1 + \left(1 + \frac{c^2 \Delta t^2}{\Delta x^2}\right) \left(1 + \frac{c^2 \Delta t^2}{\Delta y^2}\right) \left(1 + \frac{c^2 \Delta t^2}{\Delta z^2}\right)} \leq \alpha \]

\[ \leq \left( -1 + \sqrt{1 + \frac{c^2 \Delta t^2}{\Delta x^2}} \left(1 + \frac{c^2 \Delta t^2}{\Delta y^2}\right) \left(1 + \frac{c^2 \Delta t^2}{\Delta z^2}\right) \right). \]

(15)

For the uniform meshing of \( \Delta = \Delta x = \Delta y, \) (15) is described by the stability number of \( s = \sqrt{2c \Delta t / \Delta} \) for the 2D case as below,

\[ \frac{4}{s^2} - 1 \leq \alpha \leq \frac{1}{s^2}. \]

(16)

Note that increasing the stability number decreases the available region of \( \alpha \) and only \( \alpha = 0, \) i.e., the ADI-FDTD method, is unconditionally stable, regardless of the choice of \( s \) value. However, the careful choice of \( \alpha \) in (16) can generate better performances over the ADI-FDTD method. This is a compromise by the addition of the perturbed terms for better numerical performances. The numerical dispersion relation for (14) can be derived as,

\[ \cos^2(\omega \Delta t / 2) = \frac{r^2}{pq} \]

(17)

In general, the numerical phase velocity along the diagonals for the grid-based Maxwell’s equations can be smaller or larger than that along the axes, depending on the numerical conditions. The parameter \( \alpha \) can be used in order that the numerical anisotropy can be lowered within the stable region of (16). It is known that there is a specific value designated by \( \alpha_0 \) to give isotropic wave propagation for all directions by equating the phase velocity along the diagonal to that along the axes. From (17), the isotropic parameter \( \alpha_0 \) can be found as:

\[ \alpha_0 = \frac{\sqrt{(c^2 W_{00}) + 1} + 1}{2c W_{00} W_{00}} \left(1 + \left(\frac{\Delta t}{\Delta x \sin \left(\frac{k_0 \Delta x}{2}\right)}\right)^2 - 1\right) \]

(18)

where \( W_{00} = \Delta t \Delta x \left(\frac{k_0 \Delta x}{2}\right), \) \( W_{00} = \Delta t \Delta y \sin \left(\frac{k_0 \Delta y}{2}\right), \) and \( k_0 \) indicates the wavenumber in free space. The value of \( \alpha_0 \) increases slowly as the mesh density increases. We can choose the perturbed term, \( \alpha_0 \Delta \mathbf{AB}, \) such in a way that the truncation error and the numerical anisotropy can be reduced over the ADI-FDTD method.

3. NUMERICAL RESULTS

To validate the numerical dispersion relation in (17) with the optimization parameter of (16) and (18), the method is coded using (6) by the calculation steps shown in Figure 1. All numerical experiments in this article are performed in a 200 by 200 cell space. The excitation source is fed at the center of the total cell space and the numerical velocity is calculated from the time delay for propagation between two points using the matching method. A perfectly matched layer is used as an outer absorbing boundary layer. Figure 2 shows the theoretical prediction using (17) with solid lines and numerical results as markers for \( s = 1.8, \) optimized at mesh densities \( N = 40 \) and 80 cells per wavelength (CPW). The
theory and the numerical experiments agree with each other quite well. Note that there are the variations of the normalized phase velocities for various $H$ values. It can be seen that the largest numerical velocities occur along the diagonal for the chosen $H$ values, even though the velocities along the axes don’t change. We can see a directional independence of numerical phase velocities at $H = 0.1888$ and $H = 0.3$, found by (18), for $CPW = 40$ and $80$, respectively. Since the numerical velocities are independent of directions of wave propagation at the isotropic parameter values, the figure demonstrates that the anisotropy is eliminated at the calculation frequency. While the dispersion error in the isotropic propagations grows compared with the ADI-FDTD method, note the numerical dispersion can be removed to make the velocity equal to the physical value using the coefficient modification technique [7, 8]. The prerequisite of $H$ value determined by (18) must satisfy the stability condition of (16). The isotropic parameter value $H$ for increasing CPW in (18) unconditionally meets the stability condition in (16) up to $s > 1.8$. This is due to the fact that the second-order $H$ term in time step size grows for increasing time step. The isotropic parameter $H$ for $s > 1.8$ doesn’t satisfy the stability inequality of (18). Instead, $H = 1/s^2$, the right bound of the inequality, is chosen to reduce the anisotropy, maintaining the numerical stability, within the allowable range by slowing the numerical velocity along the diagonal. It can be seen in Figure 3 that the anisotropy of the proposed method are less than those of the ADI-FDTD method for $s = 1, 3$, and $5$. Though the isotropic parameter of (18) is taken for $s = 1$, $H = 1/s^2$ is used for $s = 3$ and $5$ due to the stability condition of (16). To explain the possibility that the maximum difference ratio of the numerical phase velocities along wave propagation angles can be lowered, the anisotropy is defined as

$$\text{Anisotropy} = \frac{\max[r(\phi)] - \min[r(\phi)]}{\min[r(\phi)]}$$

While the anisotropy of the ADI-FDTD is 0.0019 at $CPW = 80$ and $s = 3$, that of the proposed method is 0.0011 in the same numerical conditions. The larger the cell numbers per wavelength, the smaller the anisotropy error. Figure 4 shows the anisotropy of the ADI-FDTD and the proposed method at mesh densities from 20 to 50 CPW at $s = 3$. It can be seen that the anisotropy of the proposed FDTD is smaller than that of the ADI-FDTD. Note a decreasing anisotropy improvement for increasing CPW.
shows the anisotropy of the two methods at a fixed mesh density of 50 CPW for \( s \) numbers from 0.1 to 10. Numerical data shows that the zero-anisotropy point is \( s = 1.8 \) for the proposed method. With increasing CFL numbers, the anisotropy increases sharply for the two methods, maintaining the superiority of the proposed method.

As the comparative study of numerical computation error over the ADI-FDTD method, two 2-m-long parallel conducting plates with a separation distance of 0.02 m in free space surrounded by perfect magnetic conductor (PMC) found in literatures [2, 9] are investigated for three cases of Figure 3. Figure 6 shows \( E_y \) fields along x-axis between two plates in the steady state by the ADI-FDTD with solid lines and by the proposed method with broken lines. The field amplitudes for both methods are constant in the FDTD with solid lines and by the proposed method with broken lines. The field amplitudes of the proposed method decay more rapidly than those of the ADI-FDTD method outside the plates for all three numerical conditions. It can be seen in Figures 3 and 6 that both the anisotropy and numerical error of the proposed method is more decreased than those of the ADI-FDTD method by truncating half temporal variation differently from full temporal step in the ADI-FDTD as described in the previous section.

4. CONCLUSIONS

A two-step efficient FDTD scheme as a compromise of reduced splitting error and conditional stability is presented. It is the perturbed form of the ADI-FDTD method by the addition of second order cross derivative term. Numerical anisotropy up to \( s = 1.8 \) can be removed. While it is conditionally stable for \( s > 1.8 \), numerical anisotropy and numerical error can be decreased. It is validated from the comparison of numerical phase velocities and numerical anisotropy and numerical error can be decreased. It is the accuracy of the ADI-FDTD method, IEEE Antennas Wireless Propagat Lett 1 (2001), 31–34.

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5.8 GHZ ORIENTATION-SPECIFIC EXTRUDED-FIN HEATSink antennas for 3D RF SYSTEM INTEGRATION

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ABSTRACT: In high-power RF transmitter applications, the heatsink can be used as an antenna for improved antenna performance and increased integration. Extruded-fin heatsink antennas designed at 5.8 GHz are orientation-specific when the heatsink base replaces the patch of a patch antenna. In this case, the orientation of the fins with respect to the patch edges plays a significant role in the antenna performance and must be considered. The results show that the heatsink antenna using a lossy, low-cost FR4 substrate increases the bandwidth from 3.1 to 17.6% and radiation efficiency from 62 to 87% compared with the patch antenna on the same substrate. Also, the orientation has a significant effect on the directivity, gain, and radiation pattern. By combining two functions into one structure, the component count in a system is reduced and the antenna performance can be improved. © 2008 Wiley Periodicals, Inc. Microwave Opt Technol Lett 50: 1826–1831, 2008; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.23478

Key words: antenna efficiency; antenna gain; heatsink; high-density packaging; patch antenna; power amplifier

1. INTRODUCTION

Much research is currently being performed on 3D system-on-chip (SoC) technology. One of the main problems arises from heat dissipation. This thermal energy can affect the behavior of or compromise the devices that generate it (such as the power amplifier in the RF transceiver) and to the surrounding electronics (such as the low-noise receiver). Therefore, a heatsink or other mechanism for removing the heat is required. However, the heatsink, which is usually also electrically conductive, can affect the performance of the nearby antenna. To solve this electromagnetic